Fuzzification General Program

Professor Mohammed Zeki Khedher
Jordan University
&
Ahmed M.Z. Khedher
B.S. Math.

1. INTRODUCTION:

Lotfi Zadeh[1] is widely considered now as the first scientist to introduce the theory of fuzzy sets in 1965 although Max Black[2] had presented a paper on the theory back in 1937. Development of the fuzzy set theory has since 1965 been dramatic. Thousands of publications are now available about the basics of the fuzzy set theory as well as its applications.

Kandel[3] in 1982 gave a comprehensive bibliography of the theory.

In standard set theory, an object in either a member of a set or not. There is no in between. Traditional logics are based in the notions that P(a) is true as long as a is a member of the set belonging to class P and false otherwise.

In fuzzy set theory the degree of membership varied between 0 and 1 as will be described later.

2. BASIC DEFINITIONS OF FUZZY TERMS:

2.1 Fuzzy Predicates:

Variables or terms which do not hold very exact meaning and may be understood differently by different people. They may be even given different meaning by the same person in different circumstances or different times. Such predicates are like: expensive, safe, old, rare dangerous, educated, tall, heavy, light, smooth, rough, barable, beautiful, etc[4].

2.2 Fuzzy Quantifiers:

Quantitative terms which when added to measurable quantities may be considered fuzzy predicates e.g. many, few, almost all, usually, almost nobody, almost everybody etc.

2.3 Fuzzy Truth Values:

Grades of truthhood or falsehood can be put in a set of level e.g. extremely true, quite true, very true, almost true, more or less true, mostly true, mostly false, more or less false, almost false, very false, quite false, extremely false.. etc.

2.4 Fuzzy Modifiers:

They are the terms related to likelihood of the happening of event e.g. likely, extremely unlikely, almost impossible etc.

The above terms used in fuzzy truth values and fuzzy modifiers like very, extremely, more or less etc. are called hedgers.

2.5 Fuzzy relational operators:

In comparing two qualities in a fuzzy way, terms like approximately equal, slightly greater than, much greater than, much less than etc.

3. BASIC FUZZY SETS RELATIONS:

3.1 Definitions:

Let X be the universe of objects with elements x, where A is called a fuzzy sub-set of X (generally called a fuzzy set).

In a classical set A, the membership of x can be considered as a characteristic function μ_A from X to {0,1} such that[5]:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \quad A \\ 0 & \text{if } x \notin A \end{cases}$$

For a fuzzy set A of the universe X, the grade of membership of x in A is defined as:

$$\mu_A(x) \quad [0,1]$$

where $\mu_A(x)$ is called the membership function.

The value of $\mu_A(x)$ can be anywhere from 0 to 1. As $\mu_A(x)$ is nearer to 1.0, then x belongs to A more.

Fuzzy set elements are ordered pairs giving the value of a set element and the grade of membership *i.e.*:

$$A = \{ (x, \mu_A(x)) \mid x \quad X \}$$

Fuzzy sets are called equal if $\mu_A(x) = \mu_B(x)$ for every element x X and is denoted as:

$$A = B$$

Fuzzy sets A and B are not equal ($\mu_A(x) \neq \mu_B(x)$ for at least one x $\notin$ X) and is written as:

$$A \neq B$$

3.2 Basic Fuzzy Operations

The complement of a fuzzy set $\mu_A(x)$ is given by:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

In order for any function to be considered as a fuzzy complement, it must satisfy at least the following two requirements:

1) $c(0) = 1$ and $c(1) = 0$ *i.e.* c behaves as the ordinary complement of crisp sets.

2) For all a,b [0,1] if $a < b$ then c(a) c(b) . *i.e.* c is monotonic nonincreasing.

The Following are additional desirable requirements:

3) c is a continuous function.

4) c is involutive *i.e.* $c(c(a)) = a$ for all a [0,1].

An example of general fuzzy complements that satisfy only axiomatic skeleton:

$$c(a) = \begin{array}{ll} 1 & \text{for } a \leqslant t \\ 0 & \text{for } a > t \end{array}$$

where a [0,1] and t [0,1] : t is called the threshold of c.

While the following fuzzy complement is continuous but not involutive:

$$c(a) = 1/2 \ (1 + \cos \pi a)$$

As an example for involutive fuzzy complement:

$$c_w(a) = (1 - a^w)^{1/w}$$

where w $(0, \infty)$

When w = 1 the above function becomes:

$$c(a) = 1 - a$$

Fuzzy union of two sets A & B is given in general by the function:

$$u: [0,1] \times [0,1] \longrightarrow [0,1]$$

For each element x in the universal set:

$$\mu_{A\ \ B}(x) = u(\mu_A(x), \mu_B(x))$$

Any function of this form to be qualified as a fuzzy union; it must satisfy at least the following axioms:

1) $u(0,0) = 0$; $u(0,1) = u(1,0) = u(1,1) = 1$. *i.e.* u behaves as the classical union with crisp sets.

2) $u(a,b) = u(b,a)$. *i.e.* u is commutative.

3) If $a < a'$ and $b < b'$ then $u(a,b) < u(a',b')$. *i.e.* u is monotonic.

4) $u(u(a,b),c) = u(a,u(b,c))$. *i.e.* u is associative.

An example of fuzzy union is Yager class which is defined by the function:

$$u_w(a,b) = \min\left(1,(a^w + b^w)^{1/w}\right)$$

when $w = 2$

$$u_2(a,b) = \min\left(1, \ \ a^2 + b^2 \ \right)$$

In other words:

$$\mu_{AB}(x) = \max\left(\mu_A(x), \mu_B(x)\right)$$

Fuzzy Intersection of two fuzzy sets A & B is given by the function:

$$i : [0,1] \times [0,1] \longrightarrow [0,1]$$

The function returns the membership grade of the element in the set AB , thus:

$$\mu_{A \sqcap B}(x) = i\left(\mu_A(x), \mu_B(x)\right)$$

Such function should satisfy axioms similar to those given above for union as follows:

1) *i* (1,1) ; *i* (0,1) = *i* (1,0) = *i* (0,0) = 0. *i.e.* I behaves as the classical intersection with crisp sets.

2) *i* (a,b) = *i* (b,a). *i.e.* *i* is communicative.

3) If a < a′ and b < b′ then *i* (a,b)   < *i* (a′,b′). *i.e.* *i* is monotonic.

4) *i* (*i* (a,b),c) = *i* (a,v (b,c)). *i.e.* *i* is associative.

In other words:

$$\mu_{A \sqcap B}(x) = \min(\mu_A(x), \mu_B(x))$$

A useful fuzzy binary operation is defined as:

$$R = \{ (x,y,\ \mu_R(x,y)) \mid x \quad X, y \quad Y \}$$

For a fuzzy relation R, there is the following fuzzy computation:

$$\mu_r(y) = \sup_{yY} (\min(\mu_R(x), \mu_R(x,y)))$$

3.3 Fuzzy Measures

Fuzzy measures are criteria for measuring attributes of objects. When a certain set X is considered, the function g that makes subsets E and F correspond to the values in the interval [0,1] are called fuzzy measures if they have the following properties:

1) Boundary conditions: $g(\phi) = 0$ and $g(X) = 1$

2) Monotonicity: For every A, B   P(X)   if   A   B   then g(A) < g(B)

3) Continuity: For every sequence (A_i P(X) | *i*   N) of subset of X, if either A_1 A_2 ....   or A_v A_2 ....

(*i.e.* the sequence is monotonic) then:

$$\lim_{i \to \infty} g(A_i) = g(\lim_{i \to \infty} A_i)$$

Fuzzy measures are monotonic set functions. When property (2) satisfies the equality condition, g is referred to as a possibility measure.

4. FUZZIFICATION PROCEDURE:

The problem related to use of fuzzy set theory and fuzzy logic may be classified into three types:

1. A problem which converts exact logic problem into a fuzzy logic problem. Such problem is sometimes called fuzzification.
2. A problem which deals with originally a fuzzy logic nature of problem and converts it into a fuzzy logic problem.
3. A fuzzy logic problem which is converted to exact logic problem. Such problem is sometimes called Defuzzification.

This paper only deals with problems of first type.

4.1 Steps for fuzzification:

The following procedure is usually followed when using fuzzy set theory:

1. Description of the problem in an acceptable mathematical form.
2. Definition of the threshold for the variables, specifically the two extremes of the greatest and least degree of satisfaction.

3. Based on the above threshold values a proper membership function is selected among those available e.g. linear, piece-wise linear, trapezoidal, parabolic... etc.

4. Selection of the fuzzy operations so that the results obtained are similar to those obtained by experts.

4.2 Fuzzification general purpose program:

The purpose of this program is to prepare a ready made general purpose procedures for fuzzification & defuzzification. Only the first part is going to be presented in this paper. The program is written using Visual Basic language operating under windows. This will enable the possibilities of adding any further calculations which can be easily matched with these procedures. This will give enough portability for use with wide spectrum of problems.

4.3 General description of the program:

Two types of fuzzy relations are given:

1) piece-wise sets.

2) Continuous sets.

The two types of relation are normalized and set in general shapes so that they may be used in a straight forward manner after the necessary parameters are set to the required values.

The threshold values or rate of change of memberships:

1) piece-wise sets:

Six sets are given as shown in figure 2.

2) Continuous type sets:

Few relation are given under this type of sets as follows:

1

$$\frac{1}{1 + a\,x^2}\ , \quad \frac{1}{1 + a\,x}\ , \quad \frac{1}{1 + e^x}\ , \quad \frac{1}{1 + a\,x}\ ,$$

$$\frac{1}{(1 + a\,x)^2}\ , \quad \frac{1}{1 + a\,(x - b)^2}\ .$$

$$\delta\phi$$

Figure 2-4 show some of the windows used in the program.

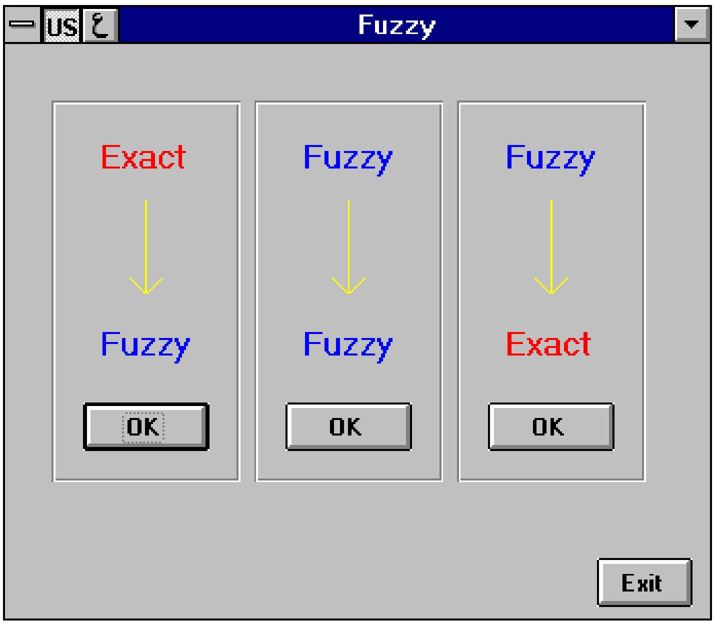

Figure 1

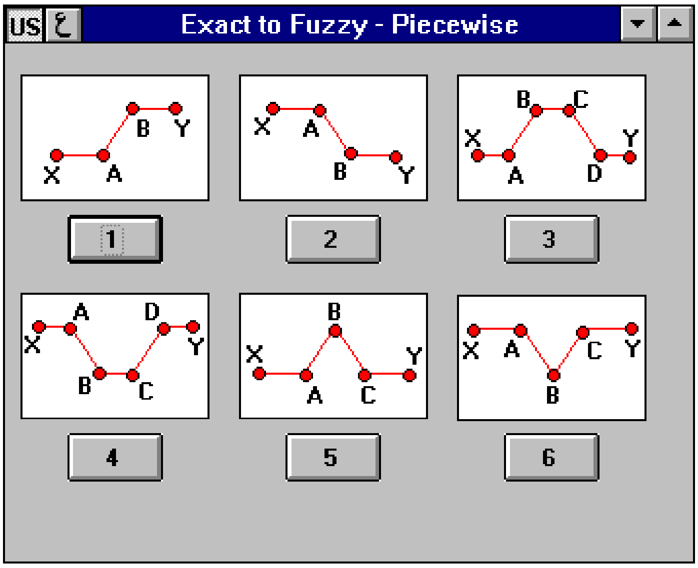

Figure 2

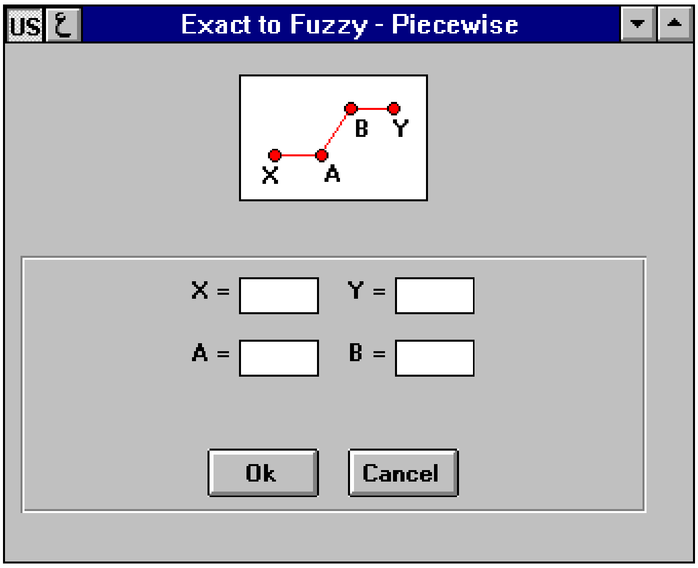

Figure 3

4. CONCLUSIONS:

A general purpose fuzzifications program in built so that non fuzzy variables are to be transferred to fuzzy ones. Various types of piece-wise and continuous relations are included. In order to use this program, program segments are to be added to it to include any further calculations necessary. The program is written using Visual Basic under Windows.

4. REFERENCES:

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